* التكامل *

- Definite Integral التكامل المحدود

* Riemann sums   * محاضرة (1)

- قواعد هامة يجب معرفتها :

① $\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n$

② $\sum_{k=1}^{n} a = a \sum_{k=1}^{n} k^0 = na$

③ $1, 2, 3, \ldots, n \longleftarrow$ متتالية حسابية $(d=1)$ Arithmetic sequence

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

④ $a, a^2, a^3, \ldots, a^n \longleftarrow$ متتالية هندسية Geometric sequence

$\sum_{k=1}^{n} a^k = \frac{a(1-a^n)}{1-a}$

⑤ Some series

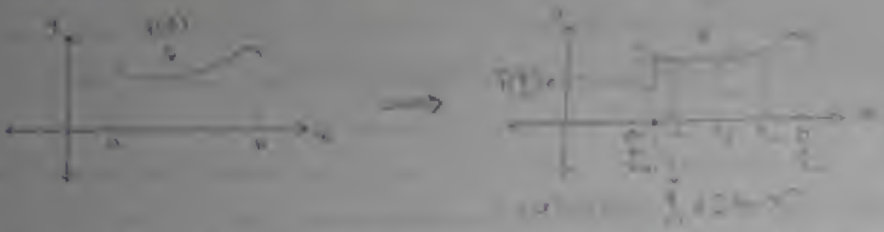
* $\sum_{1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

* $\sum_{1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

Page / Date /

→ Riemann sums

$f(x) \geqslant 0$, Continous on $[a, b]$



$\approx f(\xi_1) \cdot (x_1 - x_0)$

$\leftarrow A$

$$A \simeq \sum_{k=1}^{n} f(\xi_k) \cdot (x_k - x_{k-1}) \rightarrow ①$$

$$\text{If} \begin{cases} x_k - x_{k-1} = \dfrac{b-a}{n} \\ \xi_k = x_k \end{cases} \rightarrow ②$$

Inserting ② into ①

$\therefore$ Then

$$A \simeq \sum_{k=1}^{n} f(x_k) \cdot \frac{b-a}{n} \rightarrow ③$$

where $x_1 - \underset{=a}{x_0} = \dfrac{b-a}{n}$

$$x_1 = a + \frac{b-a}{n}$$

$$x_2 = a + 2\,\frac{b-a}{n}$$

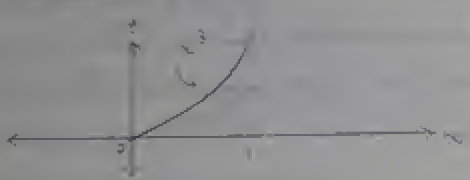
$$x_k = a + k\left(\frac{b-a}{n}\right) \rightarrow ④$$

Inserting ① into ③ To get :

(المساحة الكلية) $A = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{b-a}{n} \cdot f\left(a + k\left(\frac{b-a}{n}\right)\right)$

Ex. ① Find the area bounded by $f(x) = x^3$, $x = 0$, $y = 0$ and $x = 1$ using Riemann sums

Solution



$b = 1 \quad , \quad a = 0$

$\therefore \quad b - a = 1$

$$A = \lim_{n \to \infty} \sum \frac{1}{n} \cdot \left(0 + \frac{k}{n}\right)^3$$

$$A = \lim_{n \to \infty} \sum \frac{1}{n^4} \cdot k^3$$

$$A = \lim_{n \to \infty} \frac{1}{n^4} \sum k^3$$

$$A = \lim_{n \to \infty} \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4}$$

$$A = \frac{1}{4} \lim_{n \to \infty} \frac{n^2(n+1)^2}{n^2 \cdot n^2}$$

$$A = \frac{1}{4} \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^2$$

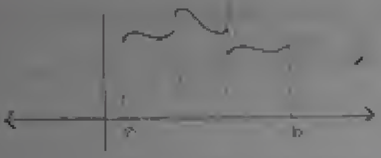
$$\boxed{A = \frac{1}{4}}$$

$\longrightarrow \int_0^1 x^3 \, dx = \frac{x^4}{4} = \frac{1}{4}$

Page/ Date/

التطبيق Riemann يجب أن تكون الدالة continuous متصلة

أي منفصلة قطعيا مثل ←

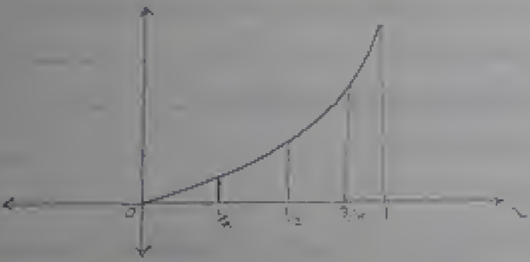


sums

Ex. ② Find the upper and lower Riemann ~~sums~~ of $f(x) = x^2$ on $[0,1]$ which correspond to the partition

تقسيم الخطوط فقط → $P_n = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$

Solution



① To get upper sums A_1

$\xi_1 = \frac{1}{4}$	$f(\xi_1) = \frac{1}{4}^2 = \frac{1}{16}$	$\frac{1}{4} - 0 = \frac{1}{4}$
$\xi_2 = \frac{1}{2}$	$f(\xi_2) = \frac{1}{2}^2 = \frac{1}{4}$	$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$
$\xi_3 = 3/4$	$f(\xi_3) = \frac{3}{4}^2 = \frac{9}{16}$	$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$
$\xi_4 = 1$	$f(\xi_4) = 1^2 = 1$	$1 - \frac{3}{4} = \frac{1}{4}$

$$A_1 = \left(\left(\frac{1}{16} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{9}{16} \times \frac{1}{4}\right) + \left(1 \times \frac{1}{4}\right) \right)$$

$$\boxed{A_1 = \frac{15}{32}}$$

To get the lower sums A_2

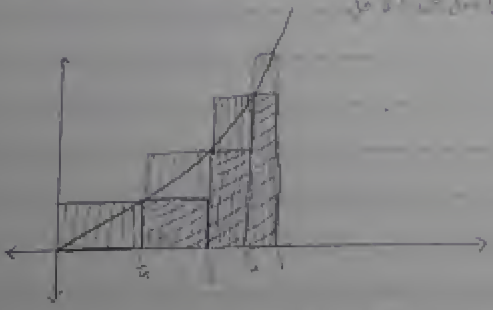
$\xi_1 = 0$	$f(\xi_1) = 0^2 = 0$
$\xi_2 = \frac{1}{4}$	$f(\xi_2) = \frac{1}{4}^2 = \frac{1}{16}$
$\xi_3 = \frac{1}{2}$	$f(\xi_3) = \frac{1}{2}^2 = \frac{1}{4}$
$\xi_4 = \frac{3}{4}$	$f(\xi_4) = \frac{3}{4}^2 = \frac{9}{16}$

$$A_2 = 0 + \left(\frac{1}{16} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{9}{16} \times \frac{1}{4}\right)$$

$$\boxed{A_2 = \frac{7}{15}}$$

[illegible]

[illegible]



The lower ← [illegible]

The upper ← [illegible]

المساحة الحقيقية

$$A_2 \leq A \leq A_1$$

Page / Date /

Ex. 3 To You

Evaluate the area bounded by
$f(x) = 3x^2$, $x = 1$, $x = 3$ and x axis

using Riemann sums

solution = 26